Interpretation of the Surface Renewal Model Through the Prandtl Mixing Length Theory

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The surface renewal model (*1*, *2*) for turbulent mixing processes has been shown by Toor and Marchello (*3*) to be applicable to turbulent pipe flow. From experimental considerations, Toor and Marchello conclude that the mixing coefficient S is related to the fluid dynamics and geometry of the problem by

$$S \propto \frac{U}{D} \cdot [N_{Re_D}]^{0.8} \tag{1}$$

The mixing coefficient S represents the rate of renewal of macroscopic fluid lumps at the wall surface, which is brought about by turbulent mixing in the fluid.

For the turbulent flow of a fluid over a flat surface, one would expect the mixing or renewal between the fluid bulk state and the wall to be controlled by the velocity fluctuations normal to the wall (v^1) and the Prandtl mixing length (l). We may therefore write

$$S \propto \frac{|v^1|}{l} \tag{2}$$

From Prandtl's mixing length theory (*4*), we know that

$$\left.\begin{aligned} |v^1| &= \text{constant} \cdot |u^1| \\ \text{and} \quad l\frac{d\bar{u}}{dy} &= u^1 \end{aligned}\right\} \tag{3}$$

Substitution for Equation (3) in Equation (2) yields

$$S \propto \frac{d\bar{u}}{dy} \tag{4}$$

Now, $(d\bar{u})/(dy)$ in Equation (4) is considered at the wall, since the turbulent exchange is between the bulk flow and the wall. At the wall $(d\bar{u})/(dy) = (\tau_0 g_c)/(\mu)$, and τ_0 for the turbulent flow over a flat surface can be expressed as

$$\tau_0 = \frac{f}{2} \cdot \rho \frac{U^2}{g_c}$$

and $f/2 = \text{constant} \cdot [N_{Re_x}]^{-0.2}$. Hence, $(g_c\tau_0)/(\mu) = \text{constant. } U/x \cdot [N_{Re_x}]^{0.8}$. Therefore,

$$S \propto \frac{U}{x} \cdot [N_{Re_x}]^{0.8}$$

By interpretating the mixing coefficient S through the Prandtl mixing length theory, the dependence of S on the fluid dynamics and geometry involved in the exchange process has been shown for the turbulent flow over a flat surface. Extension to the pipe flow problem of (*3*) can readily be carried out.

NOTATION

D = tube internal diameter, L
$\left(\frac{f}{2}\right)$ = friction factor, dimensionless
g_c = gravitational constant, dimensionless
l = Prandtl mixing length, L
N_{Re_D}; N_{Re_x} = Reynolds number based on tube diameter and surface length, respectively, dimensionless
S = mixing or renewal coefficient, $1/t$
$\bar{u}$ = mean flow velocity parallel to flat surface, L/t
u^1 = instantaneous velocity fluctuation of the flow in direction parallel to flat surface, L/t
v^1 = instantaneous velocity fluctuation of the flow in direction perpendicular to flat surface, L/t
x = distance along flat surface from the leading edge, L
y = distance perpendicular to flat surface, L
ρ = fluid density, M/L^3
μ = fluid absolute viscosity, M/Lt
τ_0 = shear stress at wall surface, M/Lt^2

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A Criterion for Fully Developed Flow of Polymer Melts in a Circular Tube

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Capillary rheometers have been used to evaluate the rheological properties of all types of fluids for many years. In order for the data obtained from such instruments to be meaningful, it is necessary that fully developed flow be achieved within the capillary. In the case of nonelastic fluids, the entrance length required for the attainment of fully developed flow is generally accepted as being

$$L_e = cDN_{Re} \tag{1}$$

The value of c, as given by various authors, ranges between 0.02 and 0.06.

On the basis of this result, it was for a long time believed that the entrance length for polymer melts would be quite small, since in most cases melt flow occurs at very low Reynolds numbers. Work done on the determination of melt die swell in recent years indicates that this is not the case. On the basis of this work, it has become recognized that the entrance length required for